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Finite-size calculation of the dynamical critical exponent of the three-state Potts model

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Abstract. The dynamical critical exponent z is calculated using the finite size scaling method for the two-dimensional three-state Potts model with non-conserved dynamics. The value of z is found to be between 2.1-2.3, with the most likely value being 2.2.

1. Introduction

Recently, the dynamics of the two-dimensional three-state Potts model has received considerable attention. Forgacs *et al* (1980) calculated the dynamical critical exponent z for the two-dimensional three-state Potts model on a square lattice. They used a Migdal type of recursion method and obtained a value of 2.25 for z. Later Binder (1981) calculated the dynamic exponent $\Delta = \upsilon z$ as $\Delta \approx 1.9$ for the two-dimensional q-state Potts model on a square lattice by analysing the relaxation of the order parameter in a Monte Carlo simulation and using dynamical scaling. His results imply that Δ does not seem to depend on the number of states q. The value $\Delta \approx 1.9$ corresponds to $z \approx 1.9$ for the Ising model ($q = 2$) and $z \approx 2.28$ for the three-state Potts model. Tobochnik and Jayaprakash (1982) found z as $z = 2.7 \mp 0.4$ for the two-dimensional three-state Potts model on a square lattice using the dynamical Monte Carlo renormalisation group (DMCRG) method. Recently, Aydın and Yalabık (1984b) evaluated the dynamical critical exponent for the same model using a modified form of the DMCRG method (Jan and Stauffer 1982, Aydın and Yalabık 1984a). They found exactly the same value obtained by Tobochnik and Jayaprakash.

In the present study, z is calculated for the two-dimensional three-state Potts model with non-conserved dynamics, using the finite size scaling method (Yalabık and Gunton 1979, Nightingale and Blöte 1980). The value of z is found to be between 2.1-2.3, with the most likely value being 2.2.

In § 2, the method and procedure are presented. The results and discussion are given in § 3.

2. The method and procedure

The Potts model can be defined through the Hamiltonian H of the form

$$-\frac{H}{kT} = K \sum_{\langle ij \rangle} \delta_{S_i S_j} \qquad S_i = 1, 2, 3 \tag{1}$$

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where the summation extends over all nearest-neighbour sites, k is the Boltzmann constant, T is the temperature and S_i represent spin variables on a lattice. K corresponds to the nearest-neighbour coupling and has a critical value K_c, $K_c = \ln(1+\sqrt{3})$, which is an exact result (Potts 1952).

In this study, a standard Monte Carlo procedure (Binder 1981) is used to simulate the dynamics of finite size Potts lattices and to calculate the average time-dependent spin correlation function $C(t)$ which is given as

$$C(t) = \langle\langle S_i(t+t')S_j(t')\rangle\rangle. \tag{2}$$

Here $\langle\langle \ldots \rangle\rangle$ denotes Monte Carlo averaging over time t' and spin sites i, j. S_i and S_j correspond either to neighbouring spins (nearest-neighbour correlation) or to the same spin (self-correlation). $C(t)$ is obtained for square lattices of sizes $n = 2$, 3, 4, 5, 6, 7, 10, 16.

Initially, a random spin configuration is generated and the system relaxes to equilibrium. After equilibrium is reached, the self and nearest-neighbour spin correlation functions are calculated as a function of time for each size of system. At sufficiently long times $C(t)$ is expected to relax with the largest time constant τ of the system. In finite systems, the asymptotic behaviour of τ as a function of n, at the critical temperature of the infinite system, can be shown by (Suzuki 1977)

$$\tau \sim n^z. \tag{3}$$

The time constant τ was calculated exactly for $n = 2$ and used as a check for the Monte Carlo procedure. For larger size lattices, τ was calculated from $C(t)$ which was obtained by averaging over 5×10^6 MCS of data for the 16×16 lattice and data covering larger times for smaller lattices.

The log-log plot of τ as a function of n is given in figure 1. The statistical errors are comparable with the size of the points on the figure. The points are expected to lie on a straight line for large n with the slope being the dynamical critical exponent z. The value of z thus obtained from figure 1 is between 2.1–2.3, with the best value being 2.2. The self and nearest-neighbour correlation functions give the same value for z. Because of the limits imposed by the available computer facilities, the correlation functions could not be obtained for sizes larger than $n = 16$.

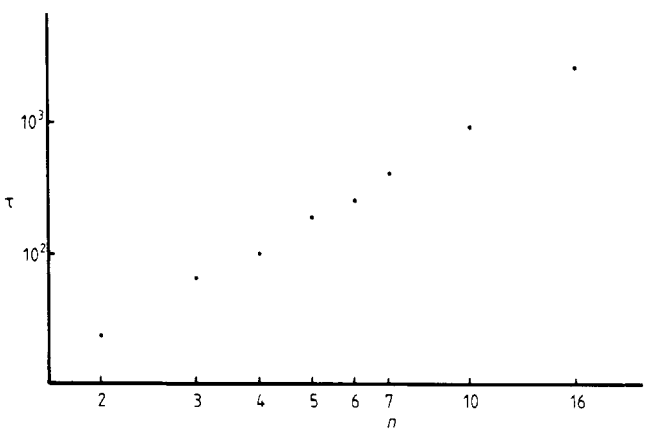

Figure 1. Log-log plot of τ against n.

3. Results and discussion

In the present study, the dynamical critical exponent z for the two-dimensional three-state Potts model is calculated using finite size scaling. The value of z is found to be between 2.1–2.3, with the most likely value being 2.2. Systematic errors (which are difficult to estimate) arising due to the possibility that we have not used large enough lattices are not represented in this interval.

Our estimate of z is consistent with the result $z = 2.25$ obtained by Forgacs *et al* (1980) and $z \approx 2.28$ calculated by Binder (1981), and different from the results of DMCRG methods. The DMCRG method used by Tobochnik and Jayaprakash (1982) gives a value for z, $z = 2.7 \mp 0.4$, which is exactly the same value found by Aydın and Yalabık (1984b). (It should be mentioned that in our DMCRG study, z is evaluated from the order parameter relaxation data over a time interval which is sandwiched between the 'early' time region and the fluctuation region at large times (Jan and Stauffer 1982). A reanalysis of our previous data with the inclusion of earlier times, which were excluded in our original study, leads to a value of z close to the Ising value.)

The main source of error in this study is not statistical, but arises due to the possibility that the system sizes may not be sufficiently large for equation (3) to hold. Hence the accuracy of the value of z we have obtained could be improved by going to larger size lattices. However, within the accuracy of this work, it is not likely that a value close to $z = 2.7$ can be obtained by this method using lattices of sizes comparable with the ones we have used.

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